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Model Uncertainty in Economic Impacts of Climate Change: Bernoulli Versus Lotka Volterra Dynamics

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ABSTRACT

The dynamic economic behavior in most integrated assessment models linking economic growth to climate change involves a differential equation solved by Jacob Bernoulli in 1695. Using the dynamic integrated climate economy (DICE) model and freezing exogenous variables at initial values, this dynamic is shown to produce implausible projections on a 60-year time frame. If world capital started at US$1, after 60 years the world economy would be indistinguishable from one starting with 10 times the current capitalization. Such behavior points to uncertainty at the level of the fundamental dynamics, and suggests that discussions of discounting, utility, damage functions, and ethics should be conducted within a more general modeling vocabulary. Lotka Volterra dynamics is proposed as an alternative with greater prime facie plausibility. With near universality, economists assume that economic growth will go on forever. Lotka Volterra dynamics alert us to the possibility of collapse. Integr Environ Assess Manag 2012;9999:xxx–xxx. © 2012 SETAC

Keywords: Bernoulli dynamics Lotka Volterra dynamics Uncertainty analysis Structured expert judgment Economic growth

INTRODUCTION

The debate on climate change is revealing, or creating, deep fault lines between the economics and physical science communities. The Fourth Assessment Report of the Intergovernmental Panel on Climate Change (IPCC) finds negative impacts on global food production and health looming above 2 °C warming. This conclusion is reiterated by the EC (2007), and the low C transition plan of the United Kingdom (DECC 2009). The 2010 Cancun Agreements, representing the consensus views of 195 United Nations member governments, propose a global target of limiting warming to 2 °C based on science. The IPCC also urges that pre-industrial atmospheric concentration of 280 ppm CO_2e should not rise above 500–550 ppm CO_2e. At the same time, economists estimating likely damages to the economy associated with a rise in temperature come to markedly different conclusions. Mendelsohn (2006) writes "…there are hardly any damages associated with a 2 °C increase in temperature." Nordhaus (2008) advocates an optimal set of policy actions that would lead to 658.5 ppm CO_2e in 2020 (associated with a temperature rise of 3.45 °C). A recent recalculation uses a higher climate sensitivity and finds a cooler optimum (Nordhaus 2011). In a dissenting voice, Weitzman (2009) emphasizes uncertainty; based on IPCC assessments of climate sensitivity, he estimates a probability on the order of 0.05 of 10 °C or more warming in 200 years (and a probability of 0.01 of 20 °C or more warming during this period) if global economic activity leads to a doubling pre-industrial greenhouse gases.

As the disagreement widens, the intramural debate between climate economists becomes less decorous. The Stern Review lead by former Chief Economist and Senior Vice-President of the World Bank, Nicholas Stern (Stern et al. 2006) is received in terms that range from "deeply flawed" (Byatt et al. 2006) to "alarmist and incompetent" (Tol 2006), to "If a student of mine were to hand in this report as a Masters thesis, perhaps if I were in a good mood I would give him a 'D' for diligence; but more likely I would give him an 'F' for fail" (Tol, BBC4). In riposte "it is surprising and regrettable that, for example, Byatt et al. (2006) and Tol and Yohe (2006) feel able to make such strong assertions on the basis of analysis that is so confused" (Dietz et al. 2007). Commenting on the integrated assessment models (IAMs) Stern finds it "very hard to believe" that IAMs "can be used as the main quantitative plank in a policy argument," because as vehicles for optimization analysis "they are still less credible" (Stern 2008). Debate has centered on IAMs linking climate variables with models of economic growth (for a review, see Parson et al. 1997; Schneider 1997; Kelly and Kolstad 1999). Damage and/or mitigation functions, temporal discounting, utility, and ethics (Stern 2008, 2009; Ackerman et al. 2009) have received critical attention. Beneath this, a fundamental Bernoulli dynamics governs the way economies develop in time. Braving the acerbic discourse, this note draws attention to model uncertainty at the level of this dynamics. The Bernoulli dynamics is shown to be implausible on the relevant timescales. The debate over damage, utility, discounting, and ethics requires a more general modeling framework; a Lotka Volterra dynamics is proposed as an alternative with more prime facie validity.

BERNOULLI DYNAMICS

Many IAMs specify economic damages as a function of temperature change, and model their impact on output and utility. The best known example is the dynamic integrated climate economy (DICE) (Nordhaus 2008), and is 1 of the models used by the Interagency Working Group on Social

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Cost of Carbon (2010). Dynamic integrated climate economy has proved to be of great value in providing a uniform yardstick for costing C emissions, and this in turn motivates efforts to enhance its realism. It is in this spirit that the underlying dynamics are reviewed. Damages $\Omega(t)$ at time t induced by temperature change $T(t)$ relative to pre-industrial mean temperature are represented in DICE as factor that reduces economic output

$$\Omega(t) = 1/[1 + 0.0028T(t)^2]. \tag{1}$$

The standard Cobb Douglas production function expresses output $Q(t)$ as a function of total factor productivity, $A(t)$ (an exogenous parameter capturing technological change), capital stock, $K(t)$, and labor $N(t)$ (also exogenous and assumed to be the population). Temperature induced damages &OHgr;(t) and abatement efforts $\Lambda(t) \in (0,1)$ reduce output:

$$Q(t) = \Omega(t)[1 - \Lambda(t)]A(t)K(t)^{\gamma}N(t)^{1-\gamma}. \tag{2}$$

Capital in the next time period is the depreciated capital of the previous time period (at rate δ), plus investment (output – consumption)

$$K(t+1) = (1-\delta)K(t) + Q(t) - C(t). \tag{3}$$

Substituting Equation 2 into Equation 3 and replacing the difference equation with a differential equation, this growth model reduces to a differential equation whose solution was given by Jacob Bernoulli in 1695 (for a derivation, see the Wikipedia page http://en.wikipedia.org/wiki/Bernoulli_differential_equation)

$$K(t) = [(1-\gamma)\int_{x=o..t} B(t)e^{-(1-\gamma)\delta x}dx + e^{-(1-\gamma)\delta t}K(0)^{(1-\gamma)}]^{1/(1-\gamma)}, \tag{4}$$

where $B(t) = \phi(t)A(t)N(t)^{1-\gamma}(1-\Lambda(t))\Omega(t)$ and $\phi(t)Q(t) = Q(t) - C(t)$.

According to Equation 3, the rate of change of capital depends only on current values in Equation 2. There is no other ''stock variable'' whose accumulation could influence capital growth. To give a prosaic example, letting $K(t)$ denote a cyclists' altitude on a mountain, does the rate of change of K depend only on the current K and the current added energy of peddling, regardless whether the cyclist is going up or down hill? Note that climate damages do not hit capital stocks directly, and capital can never decrease faster than the depreciation rate δ. Equation 4 shows that the initial capital $K(0)$ appears in the solution $K(t)$ multiplied by a negative exponential in t, hence the initial capital is ''forgotten'' with exponential speed in the solution $K(t)$.

To understand the underlying dynamics, assume there is no temperature rise, no abatement. Let exogenous variables N and A be constant at their initial values in DICE, and give δ and γ their (constant) values in DICE. Fix investment at 20% of output. Is the model's behavior plausible under these conditions? Figure 1 shows 2 capital trajectories. The solid trajectory starts with an initial capital of US\$1, that is, $\$1.5 \times 10^{-10}$ for each of the earth's 6514×10^6 people. The dotted trajectory starts with an initial capital equal to 10 times the DICE2007 initial value. The limiting capital value is independent of the starting values with a vengeance: the 2 trajectories are effectively identical after 60 years. The same will hold for any intermediate starting capital.

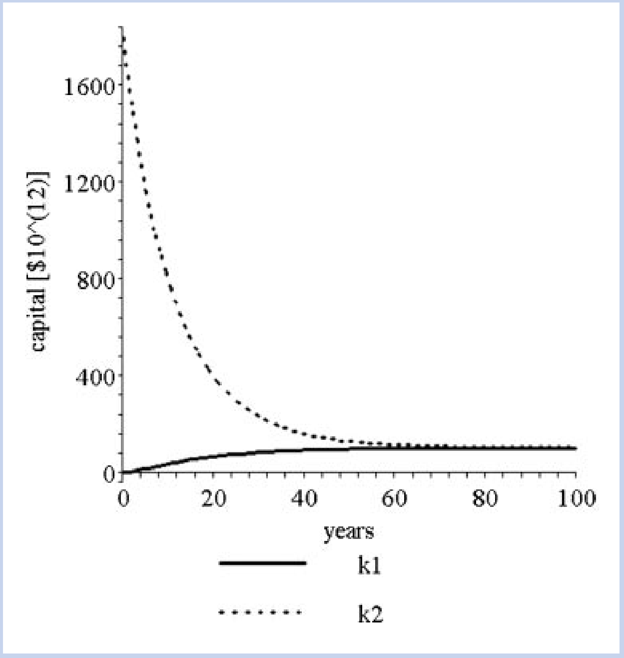

Figure 1. Two capital trajectories with DICE with default values, no temperature rise, no abatement K1(0) = US\$1 and K2(0) = \$1800 trillion.

Using the free downloadable version of DICE in XL format, one may verify the pattern in Figure 1. Figure 2 computes the capital trajectories for the paths in Figure 1 using the DICE2009XL software (DICE uses: $\delta = 0.1$, $A = 0.02722$, $N = 6514$ [10^6], $K(0) = 137[10^{12}\$]$, savings rate = 0.2, and $\gamma = 0.3$). The granularity of the 10-year timesteps delays convergence slightly.

To explore the effects of climate change in the Bernoulli dynamics (Eqn. 4), let the terms in $B(t)$ be constant, with $T(t) = T^*$. Let K^* denote the equilibrium capital level with temperature rise T^*, and K_0 the equilibrium capital level with no temperature rise ($T(t) = 0$). For $T^* = 20°C$, life as we know it on Earth would be impossible, and yet K^*/K_0 is only slightly less than one-half (0.47). That is, with constant

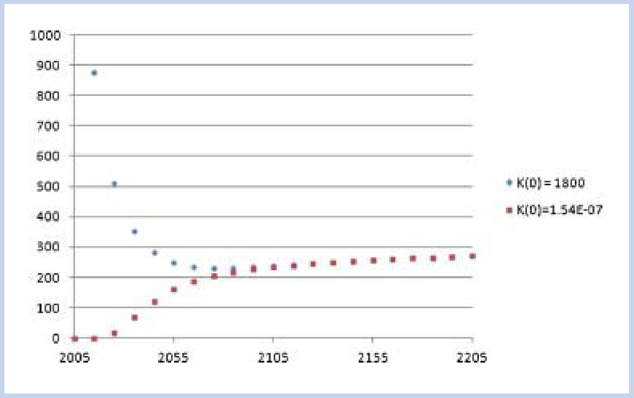

Figure 2. Output gross of abatement cost and climate damage (\$2000 trillion USD). Base case, no temperature damage, no abatement, constant population, constant total factor productivity (0.0307951), initial output from production function, and DICE defaults for other parameters (DICE 2009 EXCEL version).

population, productivity of labor, abatement effort, and investment rate, only half the capital stocks would be lost to climate damage.

Imagine that the world's capital stocks were suddenly reduced to \$1.5 × 10^{-10} per person. Humanity could not afford 1 shovel. Is it plausible that in 60 years the economic consequences would be totally effaced? At which point does Bernoulli dynamics become implausible, or better, what is the domain of empirical validity of growth models based on Bernoulli dynamics?

It is often noted that simple models like the one above cannot explain large differences across time and geography between different economies, pointing to the fact that economic output depends on many factors not present in such simple models. Barro and Sala-i-Martin (1999) note that "absolute convergence" predicts a negative correlation between growth rate of GDP per person and log(GDP) per person that is contradicted by empirical data. Instead, they advocate conditionalizing convergence on a host of covariates. Romer (2006) mentions inter alia social infrastructure, government spending, human capital, knowledge accretion, predation and protection, rent seeking, extortion, and expropriation. Interest in geographical covariates has recently been rekindled (Nordhaus 2006; Nordhaus et al. 2006; Dell et al. 2009).

If the empirical validity of the fundamental dynamics on these time sales is in question, then discussions of damage functions, utility and ethics require a broader modeling framework. Adding epicycles to the Bernoulli dynamics is not sufficient; structural alternatives should be explored.

LOTKA VOLTERRA DYNAMICS

The following simple idea suggests a Lotka Volterra dynamic: gross world production (GWP) [US\$1 trillion in 2008 dollars (USD 2008)] contributes to a stock of greenhouse gases (GHG) that, if unchecked, will eventually lead to the cessation of all productive output; with no additions to the stock of GHGs, eventually the stock recedes, where after production can resume. Pollution in the form of greenhouse gases and production are thus in a predator–prey relationship.

Greenhouse gases are modeled with the C cycle in DICE. Initially we assume (Kelly and Kolstad 2001) that emissions (Giga Tonnes Carbon, GTC) are a fixed ratio of GWP. To see where this leads, we freeze the initial emission fraction in DICE at $\varepsilon = 0.1$ [GTC/1 trillion USD 2008]. Greenhouse gases, converted to ppm CO_2e, determine the equilibrium temperature rise above pre-industrial levels according to

$$T(\mathrm{GHG}(t)) = \mathrm{cs} \times \ln(\mathrm{GHG}(t)/280)\ln(2), \qquad (5)$$

where cs is the climate sensitivity parameter (the use of equilibrium as opposed to transient temperature is a simplification that could be easily removed). Real GWP has grown at an annual rate of $\beta = 3\%$ over the last 48 years (World Bank). Dell et al. (2009) argue that rising temperature decreases the growth rate of GWP. Using country panel data, within-country cross-sectional data, and cross-country data, they derive a temperature effect that accounts for adaptation. On their analysis, yearly growth, after adaptation, is lowered by $\alpha = 0.005/°\mathrm{C}$ warming. Together with the C cycle in DICE, incorporating warming-induced damages on economic growth gives the system below. Initial values for the atmospheric GHG stock, terrestrial and shallow ocean biosphere C stock and deep ocean C stock are taken from DICE, converted from 10-year to yearly rates. Of course, the transfer coefficients in the C cycle would not remain fixed over long timescales.

$$\begin{aligned}\mathrm{GHG}(t+1) = {} & 0.988 \times \mathrm{GHG}(t) + 0.0047 \times \mathrm{Biosphere}(t)\\ & + \varepsilon \times \mathrm{GWP}(t),\end{aligned} \qquad (6)$$

$$\begin{aligned}\mathrm{Biosphere}(t+1) = {} & 0.9948 \times \mathrm{Biosphere}(t) + 0.012 \times \mathrm{GHG}(t)\\ & + 0.0001 \times \mathrm{DeepOceans}(t),\end{aligned} \qquad (7)$$

$$\begin{aligned}\mathrm{DeepOceans}(t+1) = {} & 0.9999 \times \mathrm{DeepOceans}(t) + 0.0005\\ & \times \mathrm{Biosphere}(t),\end{aligned} \qquad (8)$$

$$\mathrm{GWP}(t+1) = [1 + \beta - \alpha \times (T(\mathrm{GHG}(t)))]\mathrm{GWP}(t). \qquad (9)$$

To appreciate what this means, write the change in GWP as β GWP(t) − $\alpha \times T(\mathrm{GHG}(t)) \times \mathrm{GWP}(t)$. The increment β

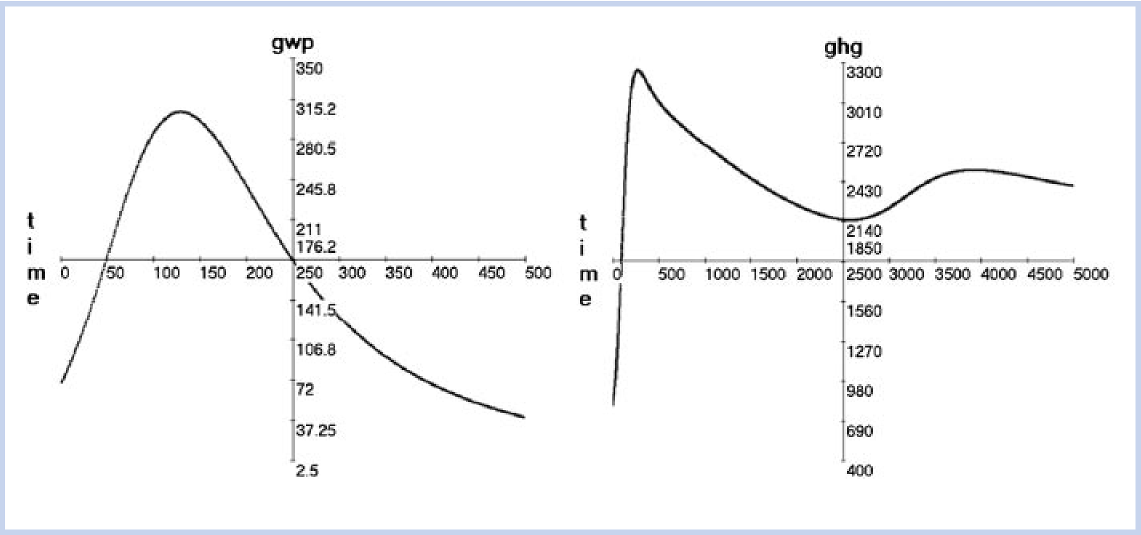

Figure 3. The impact of climate damages on GWP out to 500 years (left) and greenhouse gases [GTC] out to 5000 years (right) showing decrease after collapse of production, and increasing after 2500 when production resumes.

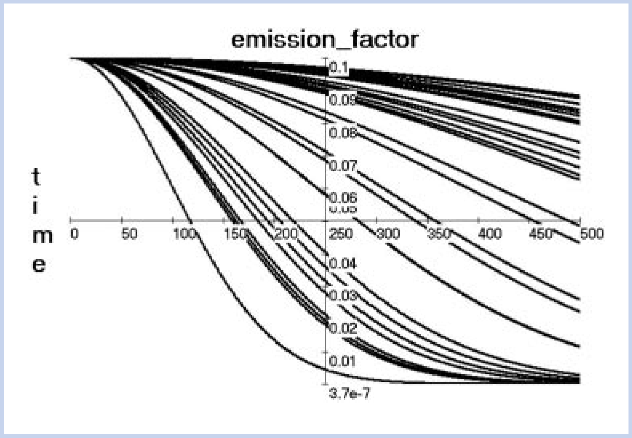

Figure 4. Thirty emission factor paths [GTC/1 trillion USD 2008].

GWP(t) is counteracted by a damage term. With the above values of β and α growth becomes negative at $T = 6°\text{C}$. If T were linear in GHG, for fixed production level the damage in GWP would be proportional to GHG, and for fixed GHG the decrement is proportional to the production level. Of course T(GHG) is not linear, but the morphology of a simple nonlinear dynamics is still at work. As T gets large, GWP declines at an increasing rate: GWP collapses. This conclusion will not surprise readers of Diamond (2005). The assumption of uninterrupted long term growth is nearly universal among economists; Lotka Volterra dynamics show that this is not the only possibility.

Figure 3 shows GWP and GHG as functions of time out to 500 years. GWP collapses. Starting at 67.79 [trillion USD], it grows to 303, and then falls back to 39.5 after 500 years.

Greenhouse gases also recede but not to their initial level. The C stock in the biosphere and the deep ocean has gone up, and these reservoirs serve as a source to the atmosphere long after industrial emissions have declined. Equilibrium is effectively reached after 10 000 years, reminding us that equilibrium values may not be relevant for current policy choices. GWP stabilizes at 7.68, never returning to the 100-year high. The equilibrium is independent of initial (positive) values of GWP, but convergence is not reached on relevant timescales. These are projections with constant emissions fraction.

Decreasing the constant emission rate ε will postpone but not prevent the GWP collapse; the humps are merely shifted to the right in Figure 3. Similarly, decreasing the damage rate α will allow us to get richer before GWP collapses; the humps get higher. A different fate within this simple model can be achieved only if, sooner or later, the emission rate effectively goes to zero. (Averting collapse could also be achieved if the damage rate went to zero, but with constant emission rate, this would lead to temperatures at which life is unsustainable.) In the DICE ''no policy'' base case, the emission fraction [GTC/GWP] goes from 0.13 to 0.011 in 200 years. The required reductions depend on new technologies whose existence is uncertain. To capture this uncertainty with a simple model, we replace ε with the time-dependent emission factor $0.1 \times \exp(-t \times \chi)$, where t is time in years, and χ is log uniformly distributed on $[10^{-6}, 10^{-4}]$. Thirty samples from the distribution of this emissions factor are shown in Figure 4.

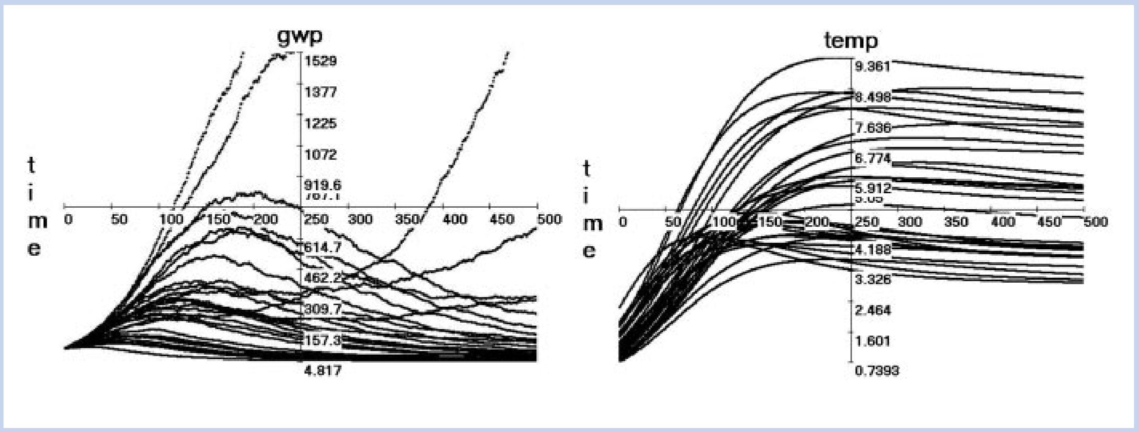

Figure 5. Thirty GWP (left) and temperature (right) paths.

Table 1. GWP after 500 years, with correlation, rank correlation, and partial correlation on dependent variables, based on 200 scenarios

	Independent variable	Pearson correlation	Rank correlation	Partial correlation	Multiple correlation
GWP	GWP growth rate	−0.037	0.011	−0.022	0.497
	Climate sensitivity	−0.204	−0.472	−0.264	
	Damage rate	−0.200	−0.518	−0.184	
	Emissions reduction rate	0.409	0.616	0.419	

GWP = gross world product.
Multiple correlation is the correlation between GWP and its best linear predictor based on the independent variables.

All paths start at 0.1, and the emission factor after 250 years ranges from 0.097 to 0.005. Of course, how these different emission paths effect GWP will depend on all the other uncertainties. Choosing ''ball park'' distributions for these parameters. Climate sensitivity is epistemic (uncertain, but constant through each run, $t = 0\ldots500$) and β distributed on [1,15] with parameters [4,24], mean 3. The damage parameter α, log uniform distributed on [0.004,0.01], and the emission rate parameter ε, log uniform on $[10^{-6}, 10^{-4}]$ are also epistemic. Figure 5 shows 30 paths for GWP and temperature.

On some paths, GWP enjoys uninterrupted growth in this time frame, on other paths GWP collapses. The timing and height at collapse depend on all uncertain parameters. Maximum temperatures range from 9.4°C to 3.7°C. The salutary growth rates in Figure 5 arise if the emission reduction rate is very aggressive, the climate sensitivity is very low, and the damage rate α is very low; and none of these factors by itself is sufficient. Table 1 shows the correlations of GWP after 500 years with GWP growth rate, climate sensitivity, damage rate, and emissions reduction rate. Of these, the most important is the emissions reduction rate. Note that the costs of the different emission reduction policies are not reflected in the GWP paths.

CONCLUSION

The debate surrounding integrated assessment models should be widened to include model uncertainty at the fundamental level of economic dynamics. Lotka Volterra dynamics alert us to the possibility that we cannot tell where we are going by looking where we have been. With constant emissions factor and defensible values from the literature for other variables, Lotka Volterra dynamics predict initial growth followed by collapse to almost half the initial GWP. With aggressive emissions reduction and good luck we may hope for continued growth in the next 500 years. Appreciating the extent of our uncertainty prioritizes uncertainty characterization and fosters humility. Continued long-term growth is not certain.

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